

CMB RADIATION POWER SPECTRUM IN CDM OPEN UNIVERSES UP TO 2nd ORDER PERTURBATIONS

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ABSTRACT

A second-order perturbation theory approach is developed to calculate temperature anisotropies in the cosmic microwave background. Results are given for open universes and fluctuations corresponding to CDM models with either Harrison-Zeldovich (HZ) or Lyth-Stewart-Ratra-Peebles (LSRP) primordial energy-density fluctuation power spectrum. Our perturbation theory approach provides a distinctive multipole contribution as compared to the primary one, the amplitude of the effect being very dependent on normalization. For low- Ω models, the contribution of the secondary multipoles to the radiation power spectrum is negligible both for standard recombination and reionized scenarios, with the 2-year COBE-DMR normalization. For a flat universe this contribution is $\approx 0.1 - 10\%$ depending on the reionization history of the universe and on the normalization of the power spectrum.

Subject headings: cosmology: cosmic microwave background - $\Omega < 1$ - 2nd order perturbations

I. INTRODUCTION

In the absence of a cosmological term ($\Lambda = 0$) and assuming standard recombination, anisotropies in the temperature of the Cosmic Microwave Background (CMB) on large angular scales ($\geq (2\Omega^{1/2})^\circ$) are generated via the Sachs-Wolfe effect ($\Omega = 1$), (Sachs and Wolfe 1967), or the generalized Sachs-Wolfe effect ($\Omega < 1$). This last case includes not only gravitational fluctuations at recombination but the integrated effect that depends on curvature of the time-varying potential along the photon trajectory (Anile and Motta 1976, Abbott and Schaefer 1986, Traschen and Eardley 1986, Gouda et al. 1991).

For smaller angular scales, there are primary contributions coming from recombination: photon fluctuations and the Doppler effect from last scattering. However, secondary anisotropies can also arise in the microwave sky due to the following physical processes: i) differential gravitational redshifts and blueshifts of the growing non-linear density fluctuations act on photons travelling to the observer that contribute an integrated gravitational effect (Martínez-González, Sanz and Silk, 1990, 1992, 1994), [†] ii) hot gas inhomogeneously distributed between recombination and the present epoch generates anisotropy associated with non-linear flows (Vishniac, 1987), and iii) dust present in the early universe, produced in the process of formation of bound structures, also generates anisotropies especially in the Wien region (Bond et al., 1991). Such effects are important if the last scattering surface is modified by reionization. An interesting possibility is that the secondary temperature fluctuations imprint signatures that differ in angular scale from the primary fluctuations (see section III).

We will focus in this paper on the first of the above mentioned effects: the generation of anisotropies by 2nd order density perturbations in an open model. For the fluctuations, we will consider a CDM model with null/negligible baryon content. Recent papers on this topic deal with the hypothesis of a flat background: 2nd order perturbations were considered by Martínez-González, Sanz and Silk (1992) whereas some (analytic) models for the non-linear evolution were treated by Martínez-González, Sanz and Silk (1994). Numerical simulations dealing with full non-linear evolution for (more realistic) models such as CDM have been performed by Tuluie and Laguna (1995). Seljak (1995) has also done numerical calculations based on N-body simulations performed by Gelb and Bertschinger (1994) for a flat universe. The generic conclusion one can extract from this work is that, at least for $\Omega = 1$, the level of anisotropy in the CMB generated at low- z by quasi-linear and non-linear evolution of the matter is $\Delta T/T \lesssim 10^{-6}$, except for some unrealistic models where growth commences at very high z .

We have computed the resulting fluctuations, which are of second order in perturbation theory relative to the uniform open cosmological background, for the case in which the density fluctuations are Gaussian. We expect non-linear contributions to further increase

[†]It is helpful to distinguish between the integrated Sachs-Wolfe effect (Sachs and Wolfe 1967; Hu and Sugiyama 1995) that occurs across linear fluctuations only when $\Omega \neq 1$ from the differential effect across clusters (Rees and Sciama 1968) that is present even if $\Omega = 1$.

these minimal fluctuations, but such effects are highly model-dependent. Our perturbation theory approach provides a relatively robust prediction: for CDM open models, $\delta T/T$ has a distinctive multipole contribution compared to the one expected in standard inflationary models at the linear level. The amplitude of the effect is very sensitive to normalization.

II. THE SECOND ORDER GRAVITATIONAL EFFECT

a) The integrated gravitational effect:

We obtained (Martínez-González, Sanz and Silk, 1990) an expression for the secondary anisotropies generated by the linear gravitational potential $\varphi(t, \vec{x})$ associated with non-linear density fluctuations $\Delta(t, \vec{x})$. For a flat or open universe with vanishing pressure,

$$(\Delta T/T)_{\text{secondary}} = 2 \int_{t_r}^{t_o} dt \frac{\partial \varphi}{\partial t}(t, \vec{x}) \quad , \quad \nabla^2 \varphi = 6\Omega a^{-1} \Delta(t, \vec{x}) \quad , \quad (1)$$

where the scale factor $a(t)$ is normalized at the present time ($a_0 = 1$) and we choose units such that $c = 8\pi G = 1$ and the Hubble length at the present time $d_o = 2H_o^{-1} = 1$. The previous equation giving the integrated gravitational effect is valid in the open case only for scales below the *curvature scale*. The line integral must be performed along the geodesic associated with the flat or open Friedmann background

$$\vec{x}(t, \vec{n}) = \lambda(a) \vec{n} \quad , \quad a(\lambda) \equiv \frac{(1 - \lambda)^2}{1 - (1 - \Omega)\lambda^2} \quad ; \quad (2)$$

where \vec{n} is the unit vector in the direction of observation. This integrated effect, except for the factor 2 that comes from general relativity, can be understood in terms of Newtonian mechanics: it represents the work performed by the photons, propagating from recombination to the present time, against the non-static gravitational potential φ .

b) Calculations:

Let us consider either an Einstein-de Sitter or open universe as background. Perturbation theory up to 2nd order, for vanishing pressure, gives the following expression for the density fluctuations:

$$\Delta(t, \vec{x}) = D\delta + D^2 \left[\frac{5}{7} \delta^2 + \vec{\nabla} \delta \cdot \vec{\nabla} \xi + \frac{2}{7} \xi_{,ij} \xi^{,ij} \right] \quad , \quad \nabla^2 \xi = \delta \quad (3)$$

where $\delta(\vec{x}) \equiv \delta_r(1 + z_r)$. The previous expression for Δ is exact up to second order for a flat model (Peebles, 1980), whereas it is a good approximation for open models with $\Omega \gtrsim 0.1$ (Bouchet et al. 1993; Catelan et al. 1995).

Consequently, the second order effect, as given by equation (1), amounts to

$$(\Delta T/T)_{secondary} = 12\Omega \int_{a_r}^{a_o} da \phi(\vec{x}(a)) \left(\frac{D}{a}\right)^2 (2f-1) \quad ,$$

where

$$\nabla^2 \phi(\vec{x}) = \frac{5}{7} \delta^2 + \vec{\nabla} \delta \cdot \vec{\nabla} \xi + \frac{2}{7} \xi_{,ij} \xi^{,ij} \quad . \quad (4)$$

In the previous equation, $D(a)$ is the growing mode of the perturbations normalized to 1 at the present time. For a flat universe: $D = a$, whereas for an open universe (Peebles, 1980):

$$D = \frac{g(x)}{g(x_o)} \quad , x = \left(\frac{1}{\Omega} - 1\right)a \quad , f(x) = \frac{d \ln D}{d \ln a} \quad ,$$

$$g(x) = 1 + \left(\frac{3}{x}\right) \left[1 + \left(1 + \frac{1}{x}\right)^{1/2} \ln((1+x)^{1/2} - x^{1/2})\right] \quad . \quad (5)$$

The basic function to be calculated for any experimental set-up is the multipole component $C_l = \langle a_{lm}^2 \rangle$ (where temperature fluctuations expanded in spherical harmonics Y_{lm} are given by $\Delta T(\vec{n}) = \sum_l \sum_{m=-l}^l a_{lm} Y_{lm}(\vec{n})$), given by the following expression for the second order gravitational effect (we shall not consider the monopole and dipole in the calculations of the temperature anisotropy below):

$$C_l = \frac{1152}{\pi} \Omega^2 \int dk k^{-2} P_{(2)}(k) R_l^2(k) \quad , R_l(k) \equiv \int_0^{\lambda_r} d\lambda \frac{1 - (1 - \Omega)\lambda}{(1 - \lambda)^2} D^2(2f-1) j_l(kp) \quad , \quad (6)$$

Here, $p \equiv \lambda/[1 - (1 - \Omega)\lambda^2]$, j_l is the Bessel function of fractional order and λ_r is the distance from the observer to the recombination surface. The function $P_{(2)}(k)$ is the power spectrum associated with the 2nd order density perturbation $\delta_2 \equiv \frac{5}{7} \delta^2 + \vec{\nabla} \delta \cdot \vec{\nabla} \xi + \frac{2}{7} \xi_{,ij} \xi^{,ij}$ and is related to the power spectrum P_ξ of the time derivative of the potential ξ by $P_\xi = \frac{1}{k^4} P_{(2)}(k)$. The second order perturbation power spectrum $P_{(2)}(k)$ is given in terms of the first order power spectrum $P(k)$ by the equation (Goroff et al. 1986, Suto and Sasaki, 1991)

$$P_{(2)}(k) = \frac{k^3}{98(2\pi)^2} \int_0^\infty dr P(kr) \int_{-1}^1 dx P(k(r^2 + 1 - 2rx)^{1/2}) \left(\frac{3r + 7x - 10rx^2}{r^2 + 1 - 2rx}\right)^2 \quad . \quad (7)$$

In the limit of small k , there is a cancellation of the three terms contributing to δ_2 , implying that $P_{(2)}(k)$ has very little power on large scales. In particular, the generic behaviour is $P_{(2)}(k) \propto k^4$ for small k , independently of the primordial power spectrum.

With regard to a possible secondary contribution coming from 3rd order density perturbations for open models (in the case of flat models this does not exist because the 1st

order gravitational potential is static), we have the following comment: the coupling of the 1st order gravitational potential $\varphi^{(1)} \propto \frac{D}{a}$ with the 3rd order potential $\varphi^{(3)} \propto \frac{D^3}{a}$ gives a kernel for the integrated gravitational effect proportional to $\frac{D^3}{a^2}(f-1)(3f-1)$, whereas the coupling of the 2nd order gravitational potential $\varphi^{(2)} \propto \frac{D^2}{a}$ with itself gives a kernel for the integrated gravitational effect proportional to $\frac{D^3}{a^2}(2f-1)^2$. This second function is always greater than the first one. Moreover, for quasi-flat models we expect a negligible contribution from the coupling of 1st-3rd order perturbations because $f \approx 1$, whereas for low- Ω models the integrated gravitational effect due to 2nd-2nd order perturbations is produced at high- z (75% of the final effect is produced in the interval $[10, 10^3]$ for $\Omega = 0.1$, see next section) where $f \approx 1$, and so there is no practically 1st-3rd order contribution. At smaller z some contribution due to the 1st-3rd coupling is produced but it is estimated to be always bounded by that due to the 2nd-2nd coupling at low- z , and this is a small fraction of the final contribution.

III. RESULTS AND CONCLUSIONS

We have applied the formalism of the previous section to calculate the predicted amplitudes of the multipole components $C_l = \langle a_{lm}^2 \rangle$. We assume an open or flat model as background and matter density perturbations corresponding to CDM models with either a primordial HZ spectrum $P(k) = Ak$ or a LSRP one (Lyth and Stewart 1990, Ratra and Peebles 1994) and $\Omega_b = 0.05$, $h=0.5$. Primary anisotropies are normalized to the 2-year COBE-DMR map as given by the analysis of Cayón et al. (1995) for HZ and Gorski et al. (1995) for LSRP. However, secondary anisotropies are normalized to $\sigma_{16} = 1$ as this effect is generated by the small scale structure, $\lesssim 100 Mpc$, where maybe the σ_{16} normalization is more appropriated (σ_{16} is the rms density fluctuation at 16 Mpc). The results of our calculations are presented in the following figures.

We plot in Figure 1 the 2nd order power spectrum $P_2(k)$, calculated according to eq.(7), for different values of Ω and normalizing to $\sigma_{16} = 1$. The amplitude of the maximum decreases for low Ω values, and appears at lower k . In Figure 2, we display C_l up to $l = 1000$ for different values of Ω due to 2nd order anisotropies (lower curves) and due to primary anisotropies (upper curves). The solid, dashed and dotted lines represent the cases $\Omega = 1, 0.3, 0.1$, respectively. For the dashed and dotted curves the results for the HZ spectrum are above the LSRP ones at $l = 100$ in all cases. With regard to the 2nd order anisotropies, the maximum is at $l \approx 250$ at the level of 10^{-13} for all Ω values ($\sigma_{16} = 1$). In the case of COBE-DMR normalization, the level of anisotropy increases by a factor 5 for $\Omega = 1$ (upper solid line) and decreases by 2 and 5 orders of magnitude for $\Omega = 0.3$ and 0.1, respectively.

We wish to emphasize that for $\Omega = 1$ more than 90% of the anisotropy produced by the second order gravitational effect is generated at a redshift $z \lesssim 10$ (Martínez-González et al. 1992). The case of $\Omega < 1$ is similar, for instance if $\Omega = 0.1$ about 80% of the effect is generated at a redshift $z < 30$ (see Figure 3). Since reionization could not have produced a substantial effect at such low redshifts, for any baryon density consistent with primordial nucleosynthesis, our results provide the *minimal* fluctuations in the CMB, independently

of the ionization history of the universe. Notice that for $\Omega = 0.1$ there appears a knee in the generation of the effect with redshift which is due to the change in the expansion of the universe from being matter dominated to curvature dominated. The low order multipoles are generated during the curvature dominated phase whereas the high order ones are generated during the matter dominated one, as can be seen in figure (3) for $l = 250$. These two contributions at relatively high and low redshift are similar to the ones appearing at the linear level, the so called SW and ISW effects.

We remark that there is no problem with the influence of non-linear scales in the second-order calculation because for any Ω the maximum of the function $R_l(k)$ (see eq.(6)), is at $k \approx l\Omega$ and the scales that are contributing to the multipole l have $k \lesssim l$, so for the calculated multipoles C_l up to $l = 1000$ only scales $\geq 6h^{-1}$ Mpc generate a 2nd order gravitational effect. Hence no contributions from density fluctuations with density contrast larger than unity enter in the perturbative calculation.

The principal conclusion of this analysis is that a deformation (*i.e.* a peak) at $l \approx 250$ in the 2D radiation power spectrum, C_l , would be a signature of the 2nd order anisotropy contribution. Detection of such a feature would allow an estimate to be made of the global parameter Ω , because the first acoustic peak emerging from the primary anisotropy is at $l(\Omega) \gtrsim 220$. However, such a deformation is negligible for low- Ω models and it is at least two orders of magnitude below the corresponding primary anisotropy in the flat case. Only a possible experiment allowing the C_l 's to be determined with an accuracy better than 1% could detect such secondary anisotropy.

On the other hand, early reionization of the universe would erase the primary anisotropy whereas the secondary one could survive with only minor changes. Figure (4) shows the primary radiation spectrum for several reionization histories ranging from standard recombination to the case of no recombination as compared to the secondary contribution for $\Omega = 1$ and HZ primordial spectrum. The secondary effect amounts to a contribution of $\sim 1 - 10\%$ in the best case.

Finally, we remark that the second order gravitational effect is very sensitive to normalization because $\Delta T/T$ is proportional to the amplitude A of the primordial spectrum.

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FIGURE CAPTIONS

- Figure 1. The second-order power spectrum for the energy-density perturbations $P_2(k)$ for $\Omega = 1, 0.3, 0.1$ is shown by the solid, dashed and dotted lines, respectively. The primordial spectrum is normalized to $\sigma_{16} = 1$.
- Figure 2. Radiation Power spectrum for the primary (upper curves) and secondary (lower curves) contributions. $\Omega = 1, 0.3, 0.1$ correspond to solid, dashed and dotted lines, respectively. The results for the HZ spectrum are above the LSRP ones at $l = 100$ in all cases. The secondary contribution is normalized to $\sigma_{16} = 1$, except for the solid upper curve which represents the flat case with the COBE-DMR normalization.
- Figure 3. Generation of the multipole $l = 250$, with maximum amplitude, for the case $\Omega = 1, 0.3, 0.1$ as a function of redshift (solid, dashed and dotted lines respectively).
- Figure 4. Radiation Power spectrum for several reionization models with $\Omega = 1$ and the HZ spectrum. Dashed lines from top to bottom correspond to the primary contribution in the following cases: standard recombination, $\tau = 0.5, 1$ and no-recombination. The solid lines represent the secondary contribution which is insensitive to the recombination history, upper curve is for COBE-DMR normalization and the lower one for $\sigma_{16} = 1$ as in figure 2.